International Release of Mathematics and Mathematical Sciences

Available at http://www.irmms.org

# Effect of Rotation on Buoyancy Driven Convection in a Liquid Layer with Insulating Permeable Boundaries

**A. K. Gupta[#1], S. K. Kalta[2]**

[#1]Professor, Himachal Pradesh University Centre for Evening Studies, Shimla-171001, India

[2]Assistant Professor, Atal Bihari Vajpayee, Govt. Institute of Engineering and Technology, Shimla-171202, India

**Email**: [#1]guptakdr@gmail.com, [2]kalta.sanjay@gmail.com.

**Abstract:**

The effect of a uniform rotation on the onset of buoyancy driven thermal convection in a horizontal layer of liquid is investigated using the classical linear stability analysis when both lower and upper bounding surfaces of the fluid layer are considered as permeable and thermally insulating. The Galerkin method is used to obtain the eigenvalue equation which is then computed numerically. Numerical results are obtained for a wide range of values of the boundary parameters characterizing the permeable nature of the boundaries. It is observed that limiting cases of the parameters include various combinations of hydrodynamic boundary conditions as the special cases. Results of this analysis indicate that the uniform rotation has the stabilizing effect on the onset of convection. The asymptotic behaviour of the critical Rayleigh number for large values of the Taylor number is also obtained.

**Keywords**: Buoyancy, Convection, Insulating, Linear stability, Rotation.

## 1. Introduction:

The stabilizing effect of rotation has been established by Chandrasekhar [1] on the onset of buoyancy driven convection for thermally conducting case of various combinations of hydrodynamic boundary conditions. The effect of rotation on convective instability induced by both surface tension and buoyancy has been studied by Namikawa et al. [2] using the classical linear stability analysis and by Gupta and Dhiman [3] using the modified linear stability analysis, for the case of thermally conducting lower rigid boundary and thermally insulating upper free boundary. Recently, the effect of uniform rotation on the onset of combined surface tension and buoyancy driven convection has been studied by Gupta and Surya [4] for thermally insulating case when the lower boundary is rigid and the upper one is free.

In this paper, we investigate the effect of rotation on the onset of buoyancy driven thermal convection in the more general frame work of the boundary conditions when both the upper and lower boundaries are thermally insulating and permeable on which the boundary conditions as specified by Beavers and Joseph [5] are applicable, using the classical linear stability analysis. The present analysis extends the work of Gupta and Kalta [6] to include the effect of uniform rotation. The Galerkin method is used to find the eigenvalue equation analytically. The numerical results obtained for a wide range of the parameters involved are presented. The results of this analysis indicate that the rotation has the stabilizing effect. Further, the critical wave number at the onset of convection is found to be zero up to a certain threshold speed of rotation and attains a non-zero value when the speed of rotation becomes larger than the threshold speed. The asymptotic behavior of the critical Rayleigh numbers for large values of Taylor numbers is obtained. A detail description of the marginal stability curves showing the influence of the uniform rotation on the onset of convection of liquid layer is also given.

## 2. Formulation of the Problem:

We consider an infinite horizontal layer of viscous and incompressible liquid of uniform thickness $d$ heated from below which is kept rotating with a constant angular velocity $\vec{\Omega}$ about an axis parallel to the direction of gravity. Both the lower and upper boundary surfaces are thermally insulating and permeable on which the boundary condition as specified by Beavers and Joseph [5] are applicable. We choose a Cartesian coordinate system of axes with the $x$ and $y$ axes of the fluid layer in the plane of the lower boundary with positive direction of the $z$ axis along the vertically upward direction so that the fluid layer is confined between the planes at $z=0$ and $z=1$. A uniform temperature gradient is maintained across the layer by maintaining the lower boundary surface at a uniform temperature $T_0$ and the upper one at temperature $T_1(\leq \mathrm{T}_0)$. Following the usual procedure for obtaining the non-dimensional form of the governing equations (Chandrasekhar [1]), are given as

$$(D^2-a^2)(D^2-a^2-p)w=\theta+\mathrm{T}^{\frac{1}{2}}D\zeta, \tag{2.1}$$

$$(D^2-a^2-pP_r)\theta=-Ra^2 w, \tag{2.2}$$

$$(D^2-a^2-p)\zeta=-\mathrm{T}^{\frac{1}{2}}Dw. \tag{2.3}$$

where $w$ is the $z$-component of the perturbation velocity, $\zeta$ is the $z$-component of vorticity, $\theta$ is the temperature perturbation, $a$ is the horizontal wave number, $P_r=\nu/\kappa$ is the thermal Prandtl number, $R=g\alpha\beta d^4/\kappa\nu$ is the Rayleigh number, $\mathrm{T}=4\Omega^2 d^4/\nu^2$ is the Taylor number with $\alpha$ as the volume coefficient of thermal expansion, $\beta=(T_0-T_1)/d$ the maintained temperature gradient, $g$ the gravitational acceleration, $\nu$ the kinematic viscosity, $\kappa$ the thermal diffusivity and $p=p_r+ip_i$ represents the growth rate of perturbations (a complex in general), $p_r$ and $p_i$ being real, $D=d/dz$. We have chosen $d$, $d^2/\nu$ and $\beta d\nu/\kappa$ as the units of length, time and temperature respectively.

Since both the lower and upper boundary surfaces are fixed and thermally insulating, the appropriate boundary conditions are:

$$w=0,\ D\theta=0,\ D\zeta-K_0\zeta=0,\ \text{at}\ z=0, \tag{2.4}$$

$$w=0,\ D\theta=0,\ D\zeta+K_1\zeta=0,\ \text{at}\ z=1. \tag{2.5}$$

Further, as both the upper and lower boundary surfaces of the liquid layer are assumed to be permeable on which the boundary conditions as specified by Beavers and Joseph [5] are applicable, given by

$$D^2w-K_0Dw=0,\ \text{at}\ z=0, \tag{2.6}$$

$$D^2w+K_1Dw=0, \text{at}\ z=1. \tag{2.7}$$

where $K_0$ and $K_1$ are non-negative dimensionless parameters characterizing the permeable nature of the lower and upper boundary respectively.
We restrict our analysis to the case when the marginal state is stationary so that the marginal state is characterized by setting $p=0$ in Equations (2.1)-(2.3) and obtain

$$(D^2-a^2)^2w=\theta+\mathrm{T}^{\frac{1}{2}}D\zeta, \tag{2.8}$$

$$(D^2-a^2)\theta=-Ra^2 w, \tag{2.9}$$

$$(D^2-a^2)\zeta=-\mathrm{T}^{\frac{1}{2}}Dw. \tag{2.10}$$

Equations (2.8) -(2.10) together with boundary conditions given by Equations (2.4) -(2.7) pose an eigenvalue problem.

**3. Solution of the Problem:**

The single term Galerkin method is convenient for solving the present problem (Finlayson [7]). Accordingly, the unknown variables $w$, $\theta$ and $\zeta$ are written as

$$w=Aw_1,\ \theta=B\theta_1 \text{ and } \zeta=C\zeta_1. \tag{3.1}$$

where $A$, $B$, $C$ are arbitrary constants and $w_1$, $\theta_1$, $\zeta_1$ are the trial functions which are chosen suitably satisfying the boundary conditions (2.4)-(2.7).

Multiplying Eq. (2.8) by $w$, Eq. (2.9) by $\theta$ and Eq. (2.10) by $\zeta$, integrating each term of the equations with respect to $z$ from 0 to 1, using the boundary conditions (2.4)-(2.7). Substituting for $w$, $\theta$ and $\zeta$ from Eq. (3.1), we obtain the following system of linear homogeneous algebraic equations:

$$\{K_0(Dw_1(0))^2 + K_1(Dw_1(1))^2 + \langle (D^2w_1)^2 + 2a^2(Dw_1)^2 + a^4w_1^2\rangle\}A - \langle w_1\theta_1\rangle B - \mathrm{T}^{\frac{1}{2}}\langle w_1 D\zeta_1\rangle C = 0, \tag{3.2}$$

$$R a^2\langle w_1\theta_1\rangle A - \langle (D\theta_1)^2 + a^2\theta_1^2\rangle B = 0, \tag{3.3}$$

$$\mathrm{T}^{\frac{1}{2}}\langle w_1 D\zeta_1\rangle A + \{K_0(\zeta_1(0))^2 + K_1(\zeta_1(1))^2 + \langle (D\zeta_1)^2 + a^2\zeta_1^2\rangle\}C = 0. \tag{3.4}$$

The system of Equations (3.2) - (3.4) will have a non-trivial solution if and only if

$$R = \frac{1}{a^2\langle w_1\theta_1\rangle^2} \times \Big[ K_0(Dw_1(0))^2 + K_1(Dw_1(1))^2 + \langle (D^2w_1)^2 + 2a^2(Dw_1)^2 + a^4w_1^2\rangle + \frac{\langle w_1 D\zeta_1\rangle^2 \mathrm{T}}{K_0(\zeta_1(0))^2 + K_1(\zeta_1(1))^2 + \langle (D\zeta_1)^2 + a^2\zeta_1^2\rangle} \Big] \langle (D\theta_1)^2 + a^2\theta_1^2\rangle. \tag{3.5}$$

where $\langle .....\rangle$ denotes integration with respect to $z$ between $z=0$ and $z=1$. We select the trial functions satisfying the boundary conditions (2.4) - (2.7) as

$$w_1 = z(z-1)\left[ z^2 - \frac{K_0K_1 + 2(3K_0 + K_1) + 12}{K_0K_1 + 4(K_0+K_1) + 12} z - \frac{2(K_1+6)}{K_0K_1 + 4(K_0+K_1) + 12)} \right], \tag{3.6}$$

$$\theta_1 = 1, \tag{3.7}$$

$$\zeta_1 = z^3 - \frac{1}{2\{K_0K_1 + 4(K_0+K_1) + 12\}}\left[ 3(K_0K_1 + 5K_0 + 3K_1 + 12)z^2 - (K_0K_1 + 6K_0)z - (K_1+6) \right]. \tag{3.8}$$

Substitution of trial functions given by Eqs. (3.6)-(3.8) into the Eq. (3.5) yields $R$ in terms of $a$, $K_0$, $K_1$ and $\mathrm{T}$ given by

$$\begin{aligned} R = {} & \frac{10}{7\{K_0K_1 + 9(K_0+K_1) + 72\}^2} \times \big[ 504\{K_0K_1 + 4(K_0+K_1) + 12\}\{K_0K_1 + 9(K_0+K_1) + 72\} \\ & + 24a^2[K_0^2(K_1^2 + 15K_1 + 72) + 3K_0(5K_1^2 + 70K_1 + 312) + 72(K_1^2 + 13K_1 + 51)] \\ & + a^4[K_0^2(K_1^2 + 17K_1 + 76) + K_0(17K_1^2 + 272K_1 + 1140) + 4(19K_1^2 + 285K_1 + 1116)] \\ & + 12[K_0^2(K_1^2 + 15K_1 + 72) + 3K_0(5K_1^2 + 70K_1 + 312) + 72(K_1^2 + 13K_1 + 51)]^2 \mathrm{T} \,/ [42\{K_0K_1 + 4(K_0+K_1) + 12\} \\ & \{K_0K_1 + 9(K_0+K_1) + 72\} + a^2[K_0^2(K_1^2 + 15K_1 + 72) + 3K_0(5K_1^2 + 70K_1 + 312) + 72(K_1^2 + 13K_1 + 51)]] \big]. \end{aligned} \tag{3.9}$$

For given values of $K_0$, $K_1$ and $\mathrm{T}$ , Eq. (3.9) gives the Rayleigh number $R$ as a function of the wave number $a$. The minimum of $R$ is the critical Rayleigh number $R_c$ and the value of $a$ at which $R$ attains minimum is the critical wave number $a_c$.

## 4. Numerical Results and Discussion:

The numerical calculations are carried out using the relation (3.9) to obtain the values of critical Rayleigh number $R_c$ and corresponding critical wave number $a_c$ for assigned values of the parameters $K_0$, $K_1$ and $\mathrm{T}$ and presented in Table 1.

From Table 1, we observe that for assigned values of the pair $(K_0, K_1)$, value of $R_c$ increases with increasing values of the Taylor number $\mathrm{T}$ , indicating that rotation has the stabilizing effect on the onset of convection. On the other hand, for a fixed value of the permeability parameter $K_0$ (or $K_1$), when $K_1$ (or $K_0$) is increased, $R_c$ increases for a given value of $\mathrm{T}$ indicating of the stabilizing effect of the permeability parameter $K_0$ (or $K_1$) on the onset of convection. Also, we find that the critical wave number at the onset of convection is zero up to a certain threshold speed of rotation and attains non-zero value when the speed of rotation becomes larger than that of threshold speed.

**Table 1**. Variation of $R_c$ and $a_c$ for various values of $K_0$, $K_1$ and $\mathrm{T}$.

| | | $\mathrm{T}=0$ | | $\mathrm{T}=10$ | | $\mathrm{T}=10^2$ | | $\mathrm{T}=10^3$ | | $\mathrm{T}=10^6$ | |
|---|---|---|---|---|---|---|---|---|---|---|---|
| $K_0$ | $K_1$ | $R_c$ | $a_c$ | $R_c$ | $a_c$ | $R_c$ | $a_c$ | $R_c$ | $a_c$ | $R_c$ | $a_c$ |
| $10^{-8}$ | $10^{-8}$ | 120.00 | 0 | 132.28 | 0 | 242.87 | 0 | 1069.46 | 2.67 | 106975 | 12.66 |
| $10^{-8}$ | $10^{-1}$ | 122.47 | 0 | 134.52 | 0 | 242.99 | 0 | 1064.15 | 2.65 | 106951 | 12.66 |
| $10^{-8}$ | 1 | 142.22 | 0 | 152.74 | 0 | 247.36 | 0 | 1027.05 | 2.45 | 106961 | 12.62 |
| $10^{-8}$ | 10 | 231.11 | 0 | 238.67 | 0 | 306.68 | 0 | 966.37 | 1.59 | 111601 | 12.56 |
| $10^{-8}$ | $10^{8}$ | 320.00 | 0 | 327.26 | 0 | 392.56 | 0 | 1044.21 | 0.84 | 123696 | 12.68 |
| $10^{-1}$ | $10^{-8}$ | 122.47 | 0 | 134.52 | 0 | 242.99 | 0 | 1064.15 | 2.65 | 106951 | 12.66 |
| $10^{-1}$ | $10^{-1}$ | 124.96 | 0 | 136.78 | 0 | 243.19 | 0 | 1058.81 | 2.62 | 106922 | 12.65 |
| $10^{-1}$ | 1 | 144.88 | 0 | 155.20 | 0 | 248.12 | 0 | 1021.42 | 2.42 | 106899 | 12.61 |
| $10^{-1}$ | 10 | 234.58 | 0 | 242.01 | 0 | 308.89 | 0 | 959.95 | 1.54 | 111413 | 12.56 |
| $10^{-1}$ | $10^{8}$ | 324.39 | 0 | 331.53 | 0 | 395.78 | 0 | 1037.45 | 0.73 | 123441 | 12.67 |
| 1 | $10^{-8}$ | 142.22 | 0 | 152.74 | 0 | 247.36 | 0 | 1027.05 | 2.45 | 106961 | 12.62 |
| 1 | $10^{-1}$ | 144.88 | 0 | 155.20 | 0 | 248.12 | 0 | 1021.42 | 2.42 | 106899 | 12.61 |
| 1 | 1 | 166.15 | 0 | 175.22 | 0 | 256.82 | 0 | 981.96 | 2.19 | 106617 | 12.57 |
| 1 | 10 | 262.54 | 0 | 269.11 | 0 | 328.19 | 0 | 915.26 | 1.08 | 110123 | 12.48 |
| 1 | $10^{8}$ | 360.00 | 0 | 366.32 | 0 | 423.22 | 0 | 992.16 | 0 | 121601 | 12.58 |
| 10 | $10^{-8}$ | 231.11 | 0 | 238.67 | 0 | 306.68 | 0 | 966.37 | 1.59 | 111601 | 12.56 |
| 10 | $10^{-1}$ | 234.58 | 0 | 242.01 | 0 | 308.89 | 0 | 959.95 | 1.54 | 11413 | 12.55 |
| 10 | 1 | 262.54 | 0 | 269.11 | 0 | 328.19 | 0 | 915.26 | 1.08 | 110123 | 12.48 |
| 10 | 10 | 392.73 | 0 | 397.39 | 0 | 439.34 | 0 | 858.86 | 0 | 109514 | 12.26 |
| 10 | $10^{8}$ | 530.53 | 0 | 534.93 | 0 | 574.59 | 0 | 971.24 | 0 | 118711 | 12.28 |
| $10^{8}$ | $10^{-8}$ | 320.00 | 0 | 327.26 | 0 | 392.56 | 0 | 1044.21 | 0.84 | 123696 | 12.68 |
| $10^{8}$ | $10^{-1}$ | 324.39 | 0 | 331.53 | 0 | 395.78 | 0 | 1037.45 | 0.73 | 123441 | 12.67 |
| $10^{8}$ | 1 | 360.00 | 0 | 366.32 | 0 | 423.22 | 0 | 992.16 | 0 | 121601 | 12.58 |
| $10^{8}$ | 10 | 530.53 | 0 | 534.93 | 0 | 574.59 | 0 | 971.24 | 0 | 118711 | 12.28 |
| $10^{8}$ | $10^{8}$ | 720.00 | 0 | 724.08 | 0 | 760.82 | 0 | 1128.16 | 0 | 127291 | 12.26 |

From Table 1, it is evident that values of $R_c$ and $a_c$ obtained here for various assigned values of $K_0$ and $K_1$ are exactly same as those obtained by Gupta and Kalta [3] in the absence of rotation, that is, when $\mathrm{T}=0$.

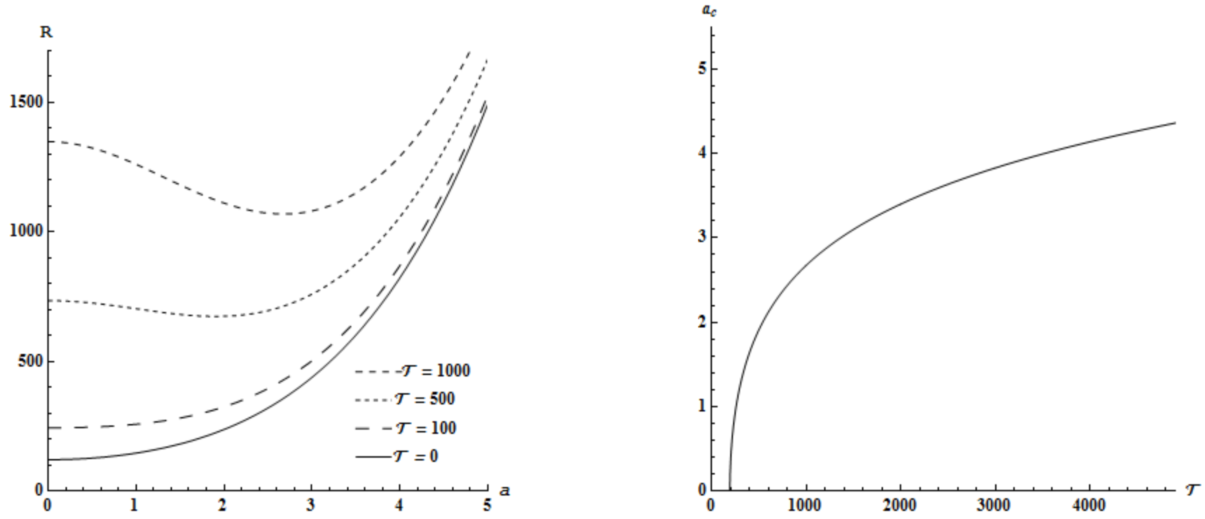
**Limiting Cases:**

The limiting cases of the parameters $K_0$ and $K_1$ in the relation (3.9) give rise to the following cases.

**Case 1.** When $K_0 \to 0$ and $K_1 \to 0$ that is, when both the lower and upper boundaries are dynamically free. In this case, we obtain $R$ from the eigenvalue Eq.(3.9) in terms of $a$ and $\mathrm{T}$ as

$$R = 120\left[1+\frac{17}{84}a^2+\frac{31}{3024}a^4+\frac{289}{168(168+17a^2)}\mathrm{T}.\right]. \tag{4.1}$$

The $(a, R)$ curves corresponding to neutral stability are plotted in Fig.1(a) for various prescribed values of $\mathrm{T}$ using the relation (4.1) showing that the increasing values of $\mathrm{T}$ has stabilizing effect at the onset of convection. The variation of the critical wave number $a_c$ with $\mathrm{T}$ at the onset of convection is illustrated in Fig.1(b). From Fig.1(b), we observe that the critical value of the Rayleigh number occurs at $a_c = 0$ up to a certain threshold value of Taylor number $\mathrm{T} \cong 195$, whereas $R_c$ occurs at a non-zero value of the wave number when $\mathrm{T}$ is greater than the threshold value.




**(a)** **(b)**

**Figure 1**: **(a)** Neutral stability curves for various values of $\mathrm{T}$ . **(b)** Variation of $a_c$ as a function of $\mathrm{T}$ .

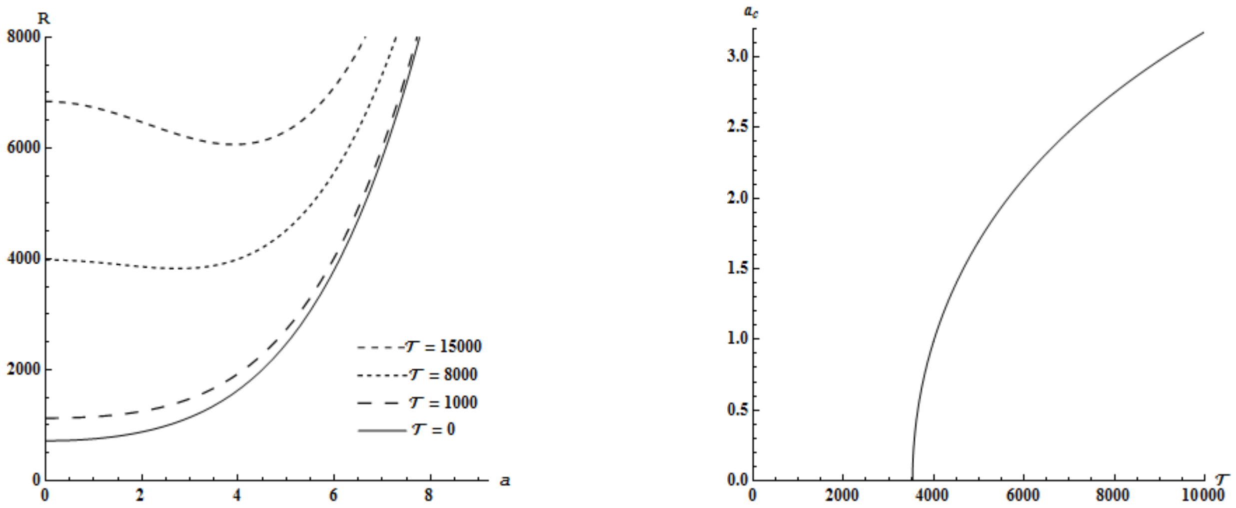
Further, the asymptotic behavior of $R_c$ and $a_c$ for large values of the Taylor number $\mathrm{T}$ obtained numerically are given by

$R_c \to 10.69\mathrm{T}^{\frac{2}{3}}$ and $a_c \to 1.3\mathrm{T}^{\frac{1}{6}}$ as $\mathrm{T} \to \infty (10^6)$.

**Case 2.** When $K_0 \to \infty$ and $K_1 \to \infty$ that is, when both boundaries are dynamically rigid.
In this case, we obtain $R$ from the eigenvalue Eq. (3.9) in terms of $a$ and $\mathrm{T}$ as

$$R = 720\left[1 + \frac{1}{21}a^2 + \frac{1}{504}a^4 + \frac{1}{42(42+a^2)}\mathrm{T}\right]. \tag{4.2}$$

The $(a, R)$ curves corresponding to neutral stability are plotted in Fig.2.(a) for various prescribed values of $\mathrm{T}$ using the relation (4.2) showing that the increasing values of $\mathrm{T}$ has stabilizing effect at the onset of convection. The variation of the critical wave number $a_c$ with $\mathrm{T}$ at the onset of convection is illustrated in Fig.2 (b). From Fig.2 (b), it is that the critical value of the Rayleigh number occurs at $a_c = 0$ up to a certain threshold value of Taylor number $\mathrm{T} \cong 3527$, whereas $R_c$ occurs at a non-zero value of the wave number when $\mathrm{T}$ is greater than the threshold value.




**(a)** **(b)**

**Figure 2**. **(a)** Neutral stability curves for various values of $\mathrm{T}$ . **(b)** Variation of $a_c$ as a function of $\mathrm{T}$ .

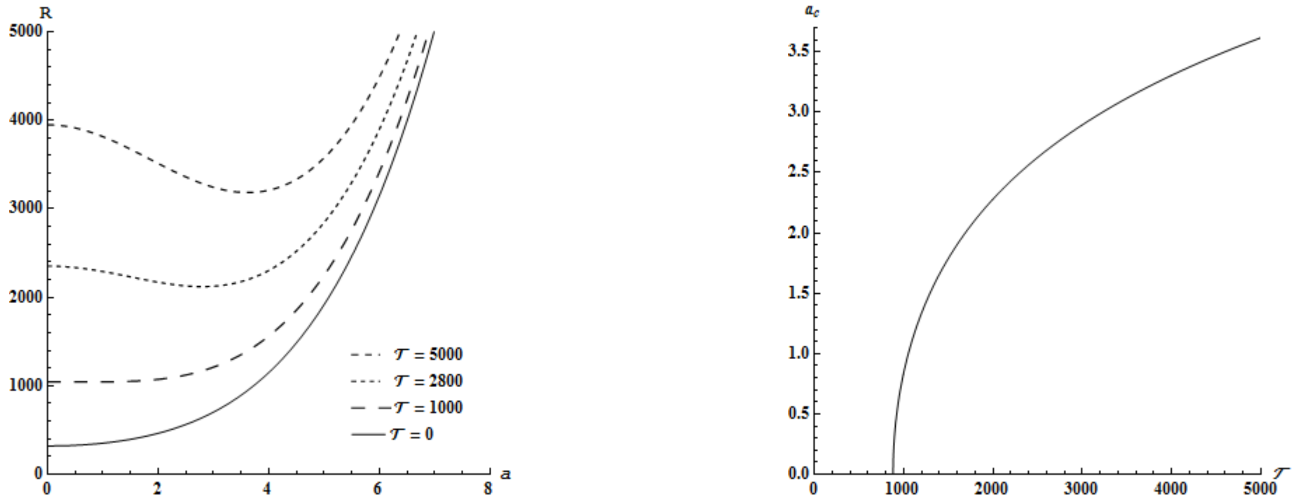
Further, the asymptotic behavior of $R_c$ and $a_c$ for large values of the Taylor number $\mathrm{T}$ obtained numerically are given by

$R_c \to 13.9\,\mathrm{T}^{\frac{2}{3}}$ and $a_c \to 1.34\,\mathrm{T}^{\frac{1}{6}}$ as $\mathrm{T} \to \infty (10^8)$.

**Case 3.** For the case when either $K_0 \to 0$ and $K_1 \to \infty$, that is, when lower boundary is dynamically free and upper one is rigid, or $K_0 \to \infty$ and $K_1 \to 0$, that is, when lower boundary is dynamically rigid and upper one is free. In either case, we obtain $R$ from the eigenvalue Eq. (3.9) in terms of $a$ and $\mathrm{T}$ as

$$R = 320\left[1 + \frac{2}{21}a^2 + \frac{19}{4536}a^4 + \frac{1}{21(21+a^2)}\mathrm{T}\right]. \qquad (4.3)$$

The $(a, R)$ curves corresponding to neutral stability are plotted in Fig.3.(a) for various prescribed values of $\mathrm{T}$ using the relation (4.3) showing that the increasing values of $\mathrm{T}$ has stabilizing effect at the onset of convection. The variation of the critical wave number $a_c$ with $\mathrm{T}$ at the onset of convection is illustrated in Fig.3 (b). From Fig.3 (b), we observe that the critical value of the Rayleigh number occurs at $a_c = 0$ up to a certain threshold value of Taylor number $\mathrm{T} \cong 881$, (approximately) whereas $R_c$ occurs at a non-zero value of the wave number when $\mathrm{T}$ is greater the threshold value.




**(a)** **(b)**

**Figure 3**. **(a)** Neutral stability curves for various values of $\mathrm{T}$. **(b)** Variation of $a_c$ as a function of $\mathrm{T}$.

Further, the asymptotic behavior of $R_c$ and $a_c$ for large values of the Taylor number $\mathrm{T}$ obtained numerically are given by

$R_c \to 10.4\,\mathrm{T}^{\frac{2}{3}}$ and $a_c \to 1.32\,\mathrm{T}^{\frac{1}{6}}$ as $\mathrm{T} \to \infty (10^8)$.

From the three limiting cases discussed above it is also observed that the obtained values of $R_c$ and $a_c$ are exactly same with those obtained by Jakeman [8] for various hydrodynamic combinations of thermally insulating boundaries when $\mathrm{T} = 0$.

**Conclusions:**

The linear stability analysis of thermal convection in the presence of rotation with insulating permeable boundaries has been studied theoretically and conclude that

1. For assigned values of the permeability parameters, rotation has the stabilizing effect on the onset of convection. It is interesting to note that value of the critical wave number at the onset of convection is found to be zero up to a certain threshold speed of rotation and attains non-zero value when the speed of rotation becomes larger than that threshold speed.
2. For a fixed value of any one of the two permeability parameters, increasing values of the other parameter has stabilizing effect on the onset of convection for a given speed of rotation.

**References:**


[1] S. Chandrasekhar, Hydrodynamic and hydromagnetic stability, London, Oxford University Press, 1961.

[2] T. Namikawa, M. Takashima and S. Matsushita, “The effect of rotation on convective instability induced surface tension and buoyancy,” J. Phy. Soc. Japan, Vol. 28, No. 5, 1970, pp. 1340-1349.

[3] A. K. Gupta and S. Dhiman, “Effect of rotation on Rayleigh-Bénard-Marangoni convection in a relatively hotter or cooler layer of liquid.” Int. J. Inno. Res. Sci. Eng. Tech. Vol. 4, 2015, No. 6, pp. 3928-3938.

[4] A. K. Gupta and D. Surya, “Effect of rotation on Rayleigh Bénard Marangoni convection in a relatively hotter or cooler layer of liquid with insulating boundaries, Asian J. Fuzzy and Applied Mathematics, Vol. 5, 2017(a), pp. 1-9.

[5] G. S. Beavers and D. D. Joseph, “Boundary conditions at a naturally permeable wall”, J. Fluid Mech., Vol. 30, 1967, pp. 197-207.

[6] A. K. Gupta and S. K. Kalta, “Buoyancy driven convection in a liquid layer with insulating permeable boundaries”, Asian J. Fuzzy and Applied Mathematics, Vol. 5, (1), 2017, pp. 10-16.

[7] B. A. Finlayson, The method of weighted residuals and variational principles, New York Academic Press, 1972.

[8] E. Jakeman, “Convective instability in fluids of high thermal diffusivity”, Physics of fluids, Vol. 11, No.1, 1967, pp. 10-14.